Journal of Engineering Physics and Thermophysics, Vol. 73, No. 4, 2000

MOTION OF A CURVILINEAR NET ON NORMAL LOCALIZED IMPACT

A. S. Kovenya[a] and M. D. Martynenko[b]

UDC 624.074.4.042.8

Within the framework of a continuous computational model, the authors obtained equations of motion for a curvilinear netted structure that in the initial state has the shape of a spherical netted dome. The motion of the net in the case of loading of one node by a normal localized impact is considered.

In [1-3], an investigation of the motion of a plane net consisting of two systems of extensible threads that in the initial state of rest formed a square net in a plane was conducted. In the present work we consider a curvilinear netted structure formed by two systems of orthogonal threads that in the initial state form a spherical netted dome. Following [1-4], we use a so-called continuous computational model that involves three groups of equations: equations of motion, geometric relationships between deformations and displacements, and relations of elasticity, in other words, equations of state of the computational model (Fig. 1).

For this, we introduce a curvilinear coordinate system of mixed type: Lagrange coordinates S_1 and S_2 and Cartesian coordinates X_1, X_2, and X_3 (global coordinates), and we consider a certain small element of the net that is a curvilinear rectangle with sides ds_1 and ds_2 and is oriented along the lines S_1 and S_2. This leads to the following equations of motion of the net element:

$$\sum_{i=1}^{2} \frac{\partial (\mathbf{N}_i \cdot \cos \beta_j^i)}{\partial s_i} + \mathbf{F}_j = \rho \frac{\partial^2 u_j}{\partial t^2} , \quad j = \overline{1, 3} , \tag{1}$$

where u_j are the displacements of the net particles; β_j^1 and β_j^2 are, respectively, the angles of rotation of the sides ds_1 and ds_2 relative to the axes of the Cartesian coordinates.

The differential relations for the deformation components can be represented as follows:

$$\frac{1}{R}\left(\frac{\partial u}{\partial \varphi_1} + w\right) = \varepsilon_1 ; \quad \frac{1}{R}\left(\frac{\partial v}{\partial \varphi_2 \sin \varphi_1} + u \operatorname{ctan} \varphi_1 + w\right) = \varepsilon_2 . \tag{2}$$

To describe displacements in the global system X_1, X_2, and X_3, we use the angles of rotation β_j^1 and β_j^2 of the sides ds_1 and ds_2, respectively, relative to the axes of the Cartesian coordinates, adding to them the angles β_j^3 of rotation of the normal ω relative to the X_1, X_2, and X_3 axes. Here the following correspondences can be established:

$$u_j = u \cos \beta_j^1 + v \cos \beta_j^2 + w \cos \beta_j^3 , \tag{3}$$

$$u = \sum_{j=1}^{3} u_j \cos \beta_j^1 ; \quad v = \sum_{j=1}^{3} u_j \cos \beta_j^2 ; \quad w = \sum_{j=1}^{3} u_j \cos \beta_j^3 . \tag{4}$$

To simplify the representation, we adopt the notation

[a]Computational-Analytical Center of the Ministry of Education of the Republic of Belarus; [b]Belarusian State University, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 73, No. 4, pp. 773-777, July–August, 2000. Original article submitted August 16, 1999.

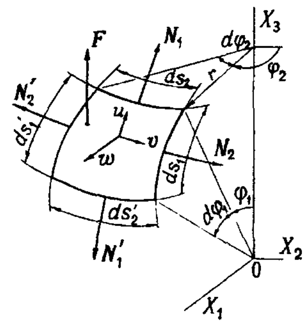

Fig. 1. Computational scheme for describing the stressed-deformed state of an element of a curvilinear net.

$$l_i = \cos \beta_1^i\,; \quad m_i = \cos \beta_2^i\,; \quad n_i = \cos \beta_3^i\,. \tag{5}$$

Thus, the investigation of the net motion is reduced to solution of the following system of differential equations:

$$a \sum_{i=1}^{2} \frac{\partial (\mathbf{N}_i \cos \beta_j^i)}{\partial \varphi_i} + \mathbf{F}_j = \rho \frac{\partial^2 u_j}{\partial t^2}\,, \tag{6}$$

$$\varepsilon_i = a \sum_{j=1}^{3} \frac{\partial u_j}{\partial \varphi_i} \cos \beta_j^i + b \sum_{j=1}^{3} u_j \cos \beta_j^1\,, \tag{7}$$

$$\mathbf{N}_i = \mathbf{N}(\varepsilon_i)\,, \tag{8}$$

$$\sum_{j=1}^{3} \cos^2 \beta_j^n = 1\,, \quad j = \overline{1,3}\,; \quad n = \overline{1,3}\,; \quad i = \overline{1,2}\,, \tag{9}$$

where $a = 1/R(\delta_{2i} \sin \varphi_1 + \delta_{1i})$; $b = \delta_{2i} \cot \varphi_1 / R$ $(i = \overline{1,2})$; δ_{ik} is the Kronecker symbol.

Differentiating Eq. (7) with respect to φ_1 and φ_2 and neglecting terms of second order of smallness, we obtain

$$\frac{\partial \varepsilon_i}{\partial \varphi_i} = a \sum_{j=1}^{3} \frac{\partial^2 u_j}{\partial \varphi_i^2} \cos \beta_j^i + b \sum_{j=1}^{3} \frac{\partial u_j}{\partial \varphi_i} \cos \beta_j^1 - c \sum_{j=1}^{3} u_j \cos \beta_j^3\,, \tag{10}$$

where $c = \delta_{2i} \cos \varphi_1 / R$.

For each of the two conditional families of threads the following relations also hold:

$$(1 + \varepsilon_i) \cos \beta_j^i = \delta_{ji} + \frac{1}{R} \frac{\partial u_j}{\partial \varphi_i}\,, \quad i = \overline{1,2}\,; \quad j = \overline{1,3}\,. \tag{11}$$

Hence, using Eq. (10), we have

$$\frac{\partial \cos \beta_j^i}{\partial \varphi_i} = \frac{1}{1 + \varepsilon_i} \left(\frac{1}{R} \frac{\partial^2 u_j}{\partial \varphi_i^2} - a \sum_{n=1}^{3} \frac{\partial^2 u_j}{\partial \varphi_i^2} \cos \beta_n^i \cos \beta_j^i - b \sum_{n=1}^{3} \frac{\partial u_j}{\partial \varphi_i} \cos \beta_n^1 \cos \beta_j^i + \right.$$

$$+ c \sum_{n=1}^{3} u_j \cos\beta_n^3 \cos\beta_j^i \Bigg). \tag{12}$$

Next, it must be taken into account [5, 6] that

$$\frac{\partial \mathbf{N}_i}{\partial \varphi_i} = \frac{d\mathbf{N}_i}{d\varepsilon_i}\frac{\partial \varepsilon_i}{\partial \varphi_i}; \quad \rho\frac{\partial^2 u_j}{\partial t^2} = E\frac{\partial^2 u_j}{\partial \varphi_i^2}. \tag{13}$$

Using expressions (10)-(13), we transform Eqs. (6)-(9) to the form

$$\frac{\partial^2 u_1}{\partial \varphi_1^2}[A_1^2 l_1^2 + F_1^2 - P_1^2(l_1^2 + l_1 m_1 + l_1 n_1)] + \frac{\partial^2 u_2}{\partial \varphi_1^2}A_1^2 l_1 m_1 + \frac{\partial^2 u_3}{\partial \varphi_1^2}A_1^2 l_1 n_1 +$$

$$+ \frac{\partial^2 u_1}{\partial \varphi_2^2}[A_2^2 l_2^2 + F_2^2 - P_2^2(l_2^2 + l_2 m_2 + l_2 n_2)] + \frac{\partial^2 u_2}{\partial \varphi_2^2}A_2^2 l_2 m_2 + \frac{\partial^2 u_3}{\partial \varphi_2^2}A_2^2 l_2 n_2 +$$

$$+ \frac{\partial u_1}{\partial \varphi_2}[B_2^2 l_1 l_2 - K_2^2(l_1 l_2 + l_2 m_1 + l_2 n_1)] + \frac{\partial u_2}{\partial \varphi_2}B_2^2 l_2 m_1 + \frac{\partial u_3}{\partial \varphi_2}B_2^2 l_2 n_1 +$$

$$+ u_1[D_2^2(l_2 l_3 + l_2 m_3 + l_2 n_3) - C_2^2 l_2 l_3] - u_2 C_2^2 l_2 m_3 - u_3 C_2^2 l_2 n_3 = \rho\frac{\partial^2 u_1}{\partial t^2}; \tag{14}$$

$$\frac{\partial^2 u_1}{\partial \varphi_1^2}A_1^2 l_1 m_1 + \frac{\partial^2 u_2}{\partial \varphi_1^2}[A_1^2 m_1^2 + F_1^2 - P_1^2(m_1^2 + l_1 m_1 + m_1 n_1)] + \frac{\partial^2 u_3}{\partial \varphi_1^2}A_1^2 m_1 n_1 +$$

$$+ \frac{\partial^2 u_1}{\partial \varphi_2^2}A_2^2 l_2 m_2 + \frac{\partial^2 u_2}{\partial \varphi_2^2}[A_2^2 m_2^2 + F_2^2 - P_2^2(m_2^2 + l_2 m_2 + m_2 n_2)] + \frac{\partial^2 u_3}{\partial \varphi_2^2}A_2^2 m_2 n_2 +$$

$$+ \frac{\partial u_1}{\partial \varphi_2}B_2^2 l_1 m_2 + \frac{\partial u_2}{\partial \varphi_2}[B_2^2 m_1 m_2 - K_2^2(l_1 m_2 + m_1 m_2 + m_2 n_1)] + \frac{\partial u_3}{\partial \varphi_2}B_2^2 m_2 n_1 -$$

$$- u_1 C_2^2 l_3 m_2 + u_2[D_2^2(l_3 m_2 + m_2 m_3 + m_2 n_3) - C_2^2 m_2 m_3] - u_3 C_2^2 m_2 n_3 = \rho\frac{\partial^2 u_2}{\partial t^2};$$

$$\frac{\partial^2 u_1}{\partial \varphi_1^2}A_1^2 l_1 n_1 + \frac{\partial^2 u_2}{\partial \varphi_1^2}A_1^2 m_1 n_1 + \frac{\partial^2 u_3}{\partial \varphi_1^2}[A_1^2 n_1^2 + F_1^2 - P_1^2(n_1^2 + l_1 n_1 + m_1 n_1)] +$$

$$+ \frac{\partial^2 u_1}{\partial \varphi_2^2}A_2^2 l_2 n_2 + \frac{\partial^2 u_2}{\partial \varphi_2^2}A_2^2 m_2 n_2 + \frac{\partial^2 u_3}{\partial \varphi_2^2}[A_2^2 n_2^2 + F_2^2 - P_2^2(n_2^2 + l_2 n_2 + m_2 n_2)] +$$

$$+ \frac{\partial u_1}{\partial \varphi_2}B_2^2 l_1 n_2 + \frac{\partial u_2}{\partial \varphi_2}B_2^2 m_1 n_2 + \frac{\partial u_3}{\partial \varphi_2}[B_2^2 n_1 n_2 - K_2^2(l_1 n_2 + m_1 n_2 + n_1 n_2)] -$$

$$- u_1 C_2^2 l_3 n_2 - u_2 C_2^2 m_3 n_2 + u_3[D_2^2(l_3 n_2 + m_3 n_2 + n_3 n_2) - C_2^2 n_2 n_3] = \rho\frac{\partial^2 u_3}{\partial t^2},$$

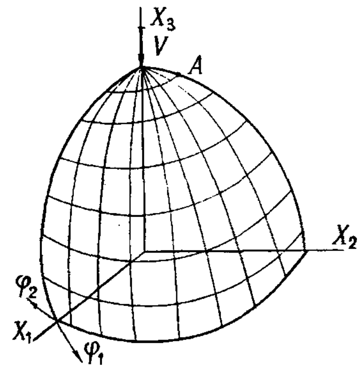

Fig. 2. Fragment of the netted region on exposure of the upper node of a dome to a normal localized impact.

where

$$A_i^2 = a^2 \frac{d\mathbf{N}_i}{d\varepsilon_i}; \quad B_i^2 = ab \frac{d\mathbf{N}_i}{d\varepsilon_i}; \quad C_i^2 = ac \frac{d\mathbf{N}_i}{d\varepsilon_i}; \quad P_i^2 = a^2 \frac{\mathbf{N}_i}{1+\varepsilon_i};$$

$$K_i^2 = ab \frac{\mathbf{N}_i}{1+\varepsilon_i}; \quad F_i^2 = \frac{a}{R} \frac{\mathbf{N}_i}{1+\varepsilon_i}; \quad D_i^2 = ac \frac{\mathbf{N}_i}{1+\varepsilon_i}, \quad i = 1, 2.$$

Next, we consider a curvilinear netted structure in the shape of a spherical netted dome with a rise $f = R$, fastened on a rigid supporting contour in the form of a ring of radius R. We assume that the rods that make up the dome are manufactured from a linearly elastic material; in this case the equations of motion of the net (14) can be written in the following form (to be specific, below we give only one equation of system (14)):

$$\frac{\partial^2 u_1}{\partial \varphi_1^2}\left[l_1^2 + \frac{\varepsilon_1}{1+\varepsilon_1}(1 - l_1^2 - l_1 m_1 - l_1 n_1)\right] + \frac{\partial^2 u_2}{\partial \varphi_1^2} l_1 m_1 + \frac{\partial^2 u_3}{\partial \varphi_1^2} l_1 n_1 +$$

$$+ \frac{\partial^2 u_1}{\partial \varphi_2^2} \frac{1}{\sin^2 \varphi_1}\left[l_2^2 + \frac{\varepsilon_2}{1+\varepsilon_2}(\sin \varphi_1 - l_2^2 - l_2 m_2 - l_2 n_2)\right] + \frac{\partial^2 u_2}{\partial \varphi_2^2} \frac{1}{\sin^2 \varphi_1} l_2 m_2 +$$

TABLE 1. Change in the State Parameters of the Node $A(\frac{\pi}{10}, 0)$ with Time (the impact velocity is 50 m/sec; $u_1 = 0$)

t, sec	u_2, m	u_3, m
$6.0 \cdot 10^{-5}$	$3.5 \cdot 10^{-12}$	$-1.3 \cdot 10^{-8}$
$9.0 \cdot 10^{-5}$	$2.2 \cdot 10^{-11}$	$-9.4 \cdot 10^{-8}$
$1.2 \cdot 10^{-4}$	$7.5 \cdot 10^{-11}$	$-3.8 \cdot 10^{-7}$
$1.5 \cdot 10^{-4}$	$1.8 \cdot 10^{-10}$	$-1.2 \cdot 10^{-6}$
$1.8 \cdot 10^{-4}$	$3.0 \cdot 10^{-10}$	$-2.8 \cdot 10^{-6}$
$2.7 \cdot 10^{-4}$	$-6.0 \cdot 10^{-10}$	$-2.1 \cdot 10^{-5}$
$3.0 \cdot 10^{-4}$	$-2.6 \cdot 10^{-9}$	$-3.6 \cdot 10^{-5}$
$3.6 \cdot 10^{-4}$	$-1.5 \cdot 10^{-8}$	$-8.8 \cdot 10^{-5}$
$3.9 \cdot 10^{-4}$	$-2.8 \cdot 10^{-8}$	$-1.3 \cdot 10^{-4}$
$4.8 \cdot 10^{-4}$	$-1.3 \cdot 10^{-7}$	$-3.7 \cdot 10^{-4}$
$6.0 \cdot 10^{-4}$	$-5.8 \cdot 10^{-7}$	$-1.1 \cdot 10^{-3}$

$$+\frac{\partial^2 u_3}{\partial \varphi_2^2}\frac{1}{\sin^2\varphi_1}l_2 n_2+\frac{\partial u_1}{\partial \varphi_2}\frac{\cos\varphi_1}{\sin^2\varphi_1}\left[l_1 l_2-\frac{\varepsilon_2}{1+\varepsilon_2}(l_1 l_2+l_2 m_1+l_2 n_1)\right]+\frac{\partial u_2}{\partial \varphi_2}\frac{\cos\varphi_1}{\sin^2\varphi_1}l_2 m_1+$$

$$+\frac{\partial u_3}{\partial \varphi_2}\frac{\cos\varphi_1}{\sin^2\varphi_1}l_2 n_1+u_1\operatorname{ctan}\varphi_1\left[\frac{\varepsilon_2}{1+\varepsilon_2}(l_2 l_3+l_2 m_3+l_2 n_3)-l_2 l_3\right]-u_2\operatorname{ctan}\varphi_1\, l_2 m_3-$$

$$-u_3\operatorname{ctan}\varphi_1\, l_2 n_3=\frac{R^2\rho}{E}\frac{\partial^2 u_1}{\partial t^2}.$$

The mixed problem obtained was solved numerically in the region (Fig. 2)

$$D=\left\{0\le\varphi_1\le\frac{\pi}{2};\ 0\le\varphi_2\le\frac{\pi}{2};\ 0\le t\le T\right\}.$$

Here, as the initial conditions, we selected ones that simulate impact against the upper node of the netted dome fastened on the rigid circular supporting contour:

$$t=0,\ 0\le\varphi_1\le\frac{\pi}{2},\ 0\le\varphi_2\le\frac{\pi}{2}:\ u_1=u_2=u_3=0;$$

$$t\ge0,\ \varphi_1=0,\ 0\le\varphi_2\le\frac{\pi}{2}:\ \frac{\partial u_1}{\partial t}=\frac{\partial u_2}{\partial t}=0;\ \frac{\partial u_3}{\partial t}=V$$

$$t\ge0,\ 0\le\varphi_1\le\frac{\pi}{2},\ 0\le\varphi_2\le\frac{\pi}{2}:\ \frac{\partial u_1}{\partial t}=\frac{\partial u_2}{\partial t}=\frac{\partial u_3}{\partial t};\tag{15}$$

$$t>0,\ \varphi_1=\frac{\pi}{2},\ 0\le\varphi_2\le\frac{\pi}{2}:\ u_1=u_2=u_3=0.$$

In particular, conditions (15) mean the following: at the instant of the action of a certain pulse load on the net, the point with the coordinate $\varphi_1 = 0$ acquires some instantaneous velocity V; in addition, it was assumed that the propagation velocity of perturbations is independent of the direction in the global coordinate system X_1, X_2, and X_3.

To solve the problem, we used a three-layer difference scheme. An analysis of the calculation results allows us to obtain the scheme of deformation of the netted structure over a certain fixed time interval. We present results of calculations for a net node located near the dome vertex (see Table 1).

In conclusion it should be noted that the method used makes it possible to evaluate the strength properties of a netted structure in a wide range of dynamic loads.

NOTATION

R, radius of curvature; r, radius of the parallel circle; **N**, linear force; ρ, specific surface density of the net; ε, relative deformation; **F**, specific surface load on the net; E, Young's modulus; β, direction angle; l, m, and n, cosines of the direction angles; V, velocity of motion of the net node; u, v, w, displacements in the local coordinate system; t, time.

REFERENCES

1. O. K. Kasumov, *Izv. Akad. Nauk AzSSR, Ser. Fiz.-Tekh. Mat. Nauk*, No. 3, 57-61 (1983).
2. D. G. Agalarov, *Izv. Akad. Nauk AzSSR, Ser. Fiz.-Tekh. Mat. Nauk*, No. 6, 38-41 (1982).

3. V. T. Aliev, *Numerical Solution of the Problem of Wave Propagation on Localized Impact against a Net Fastened on a Frame*[in Russian], Moscow (1986); Dep. at VINITI 29.12.86, No. 8940-386.
4. G. I. Pshenichnov, *Theory of Thin Elastic Netted Shells and Plates* [in Russian], Moscow (1982).
5. Kh. A. Rakhmatulin and Yu. A. Dem'yanov, *Strength in the Case of Intense Brief Loads* [in Russian], Moscow (1961).
6. Yu. N. Rabotnov, *Mechanics of a Deformable Solid* [in Russian], Moscow (1988).